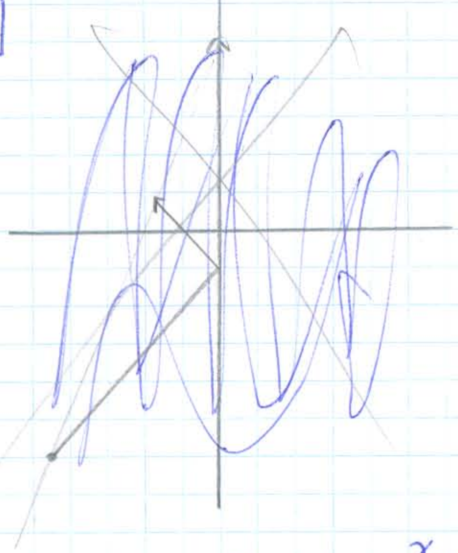
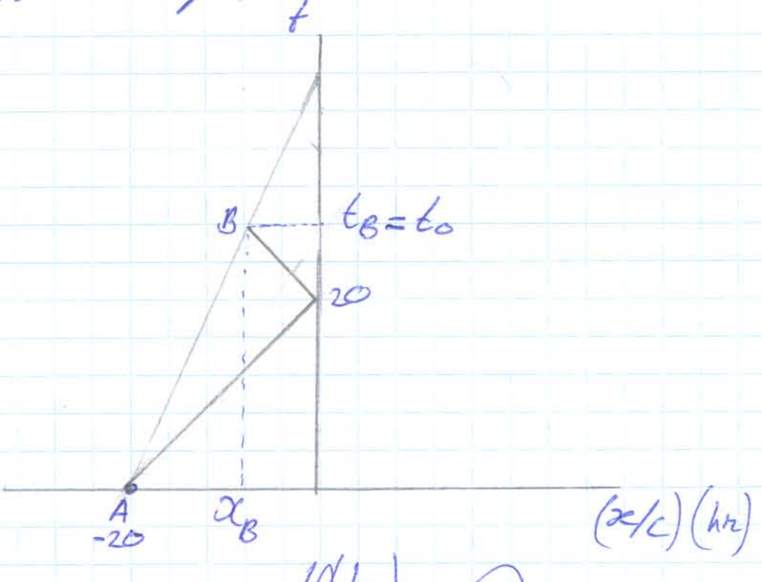
Toets Relativiteits Theorie $\boxed{24}$

Opgave 1 Landing van een Ruimte schip.

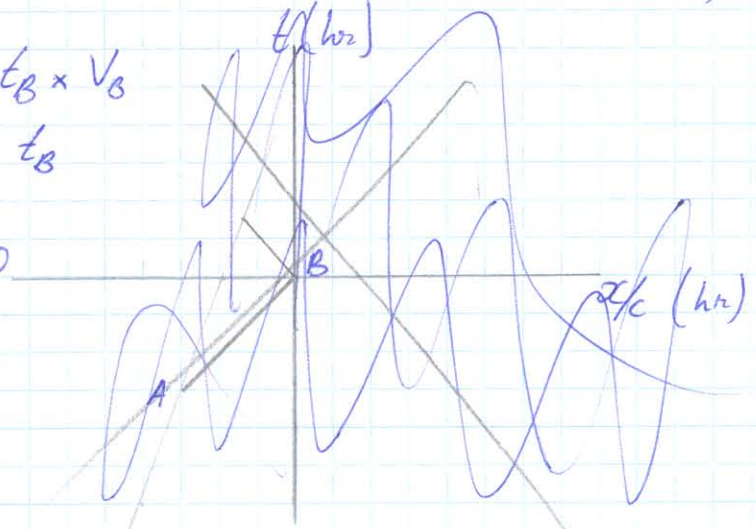
a)





$$① \quad x_B - x_A = t_B \times V_B$$

$$② \quad 20 - x_B = t_B$$



$$① \Rightarrow x_B = \frac{1}{\sqrt{2}} \cdot t_B - 20$$

$$② \Rightarrow x_B = -t_B + 20$$

$$\frac{1}{\sqrt{2}} \cdot t_B - 20 = 20 - t_B$$

$$\left(1 + \frac{1}{\sqrt{2}}\right) t_B = 40$$

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$$t_B = 40 / \left(1 + \frac{1}{\sqrt{2}}\right)$$

$$23,43 \text{ hr}$$

$$x_B = 20 - t_B = -3,43 \text{ hr}$$

$$\underline{B = (-3,43 \text{ hr} ; 23,43 \text{ hr})}$$

(hr = uur)

b) $v(t) = \frac{dx}{dt}$ $\quad x(t)$

$$= \frac{1}{\omega}\left(\cos\left(\omega(t - t_0) + \alpha_0\right) \times \omega\right)$$

$$= \cos\left(\omega(t - t_0) + \alpha_0\right) \Rightarrow 0$$

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$$\omega(t - t_0) + \alpha_0 = \pi/2 + \pi k \qquad \text{met } k \in \mathbb{Z}$$

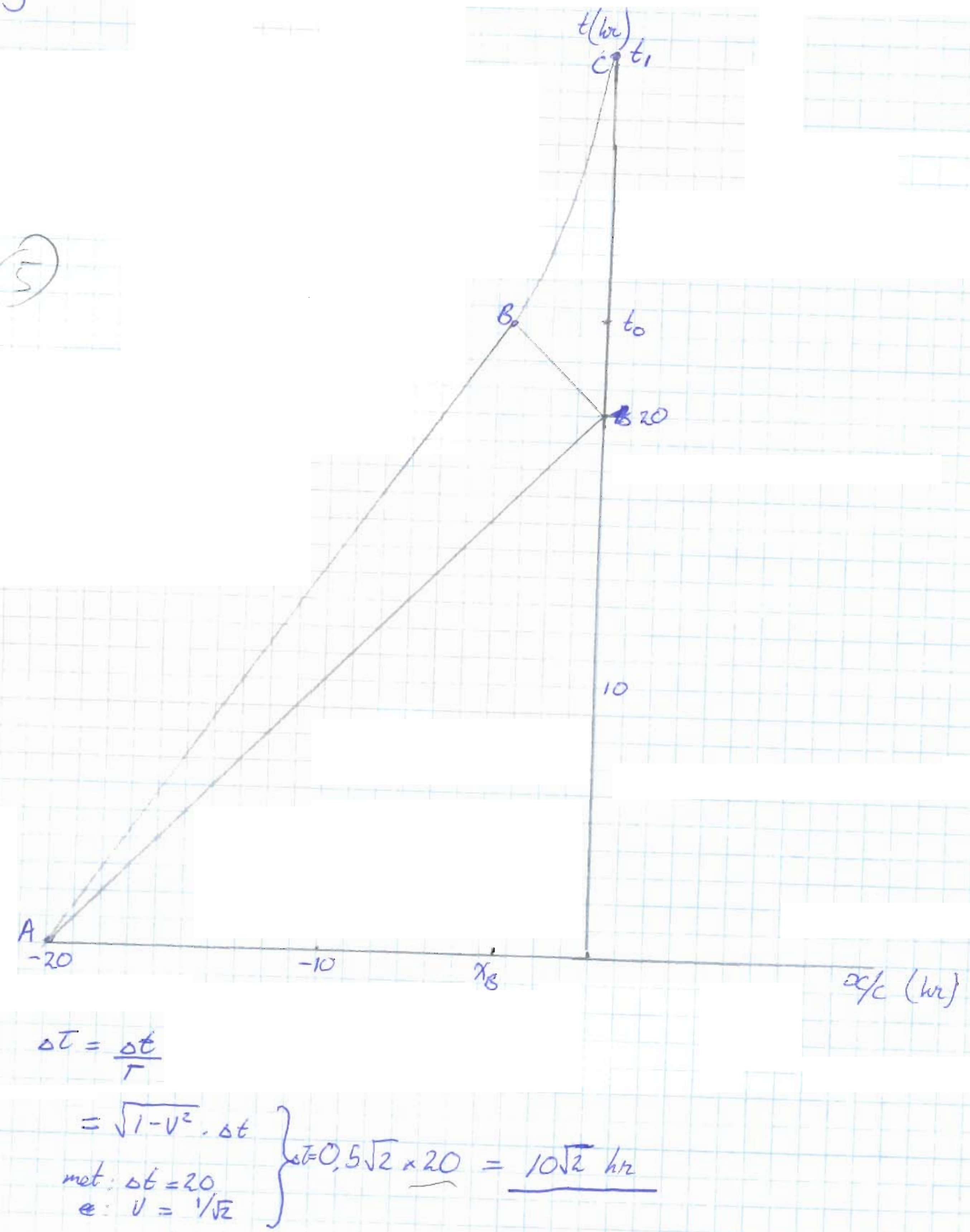
$$\omega(t - t_0) = \pi/4 + \pi k$$

$$t - t_0 = 10 + 40k \qquad \text{met } t_0 = t_B = 23,43$$

$$t = 33,43 + 40k \qquad k = 0 \text{ bij eerste landing}$$

$$\underline{t_1 = 33,43 \text{ hr}}$$

c)



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d) $\Delta\tau = \frac{\Delta t}{\gamma}$

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$$= \sqrt{1-v^2}\cdot\Delta t \quad \left.\begin{matrix} \text{met: } \Delta t = 20 \\ e: \ v = 1/\sqrt{2} \end{matrix}\right\} \Delta\tau = 0{,}5\sqrt{2}\times 20 = \underline{10\sqrt{2}\ hr}$$

e) $\Delta\tau = \int_{t_0}^{t_1} \sqrt{1-v(t)^2}\, dt \qquad v(t) = \cos[\omega(t-t_0)+\alpha_0]$ zie opgave 1b

$$= \int_{t_0}^{t_1} \sqrt{1-\cos^2[\omega(t-t_0)+\alpha_0]}\, dt$$

$$= \int_{t_0}^{t_1} \sqrt{\sin^2[\omega(t-t_0)+\alpha_0]}\, dt$$

$$= \int_{t_0}^{t_1} \sin[\omega(t-t_0)+\alpha_0]\, dt$$

$$\Delta T = -\frac{1}{\omega}\cos\left[\omega(t-t_0)+\varphi_0\right]\Big|_{t_0}^{t_1}$$

met $t_0 = 23{,}43$
en $t_1 = 33{,}43$

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$$= -\frac{1}{\omega}\left(\cos\left[\omega(t_1-t_0)+\varphi_0\right]-\cos\varphi_0\right)$$

$$= \frac{\cos\alpha}{\omega} = \underline{9{,}0\,hr}$$

Opgave 1 Lorentz contractie

a) De gecontraheerde lengt $L' = 90\,m$

$$L' = \frac{L}{\Gamma} \Rightarrow \Gamma = \frac{L}{L'}$$

$$= \cancel{90/150 = 3/5}$$

$$150/90 = 5/3$$

$$\Gamma = \frac{1}{\sqrt{1-v^2}}$$

$$\frac{1}{\Gamma^2} = 1 - v^2$$

$$\left.\begin{array}{l}\sqrt{1-\frac{1}{\Gamma^2}} = v \\ \text{met } \Gamma = \frac{5}{3}\end{array}\right\} \underline{v = \frac{4}{5}}$$

~~In SI $v = \frac{4}{5} c$ m/s~~

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b) Lengte van tunnel in Stelsel S' van de trein $= L'_T$

$$L'_T = \frac{L_T}{\Gamma} = \frac{3}{5} \times 100 = 60\,m.$$

$$150 - 60 = \underline{90\,m.}$$

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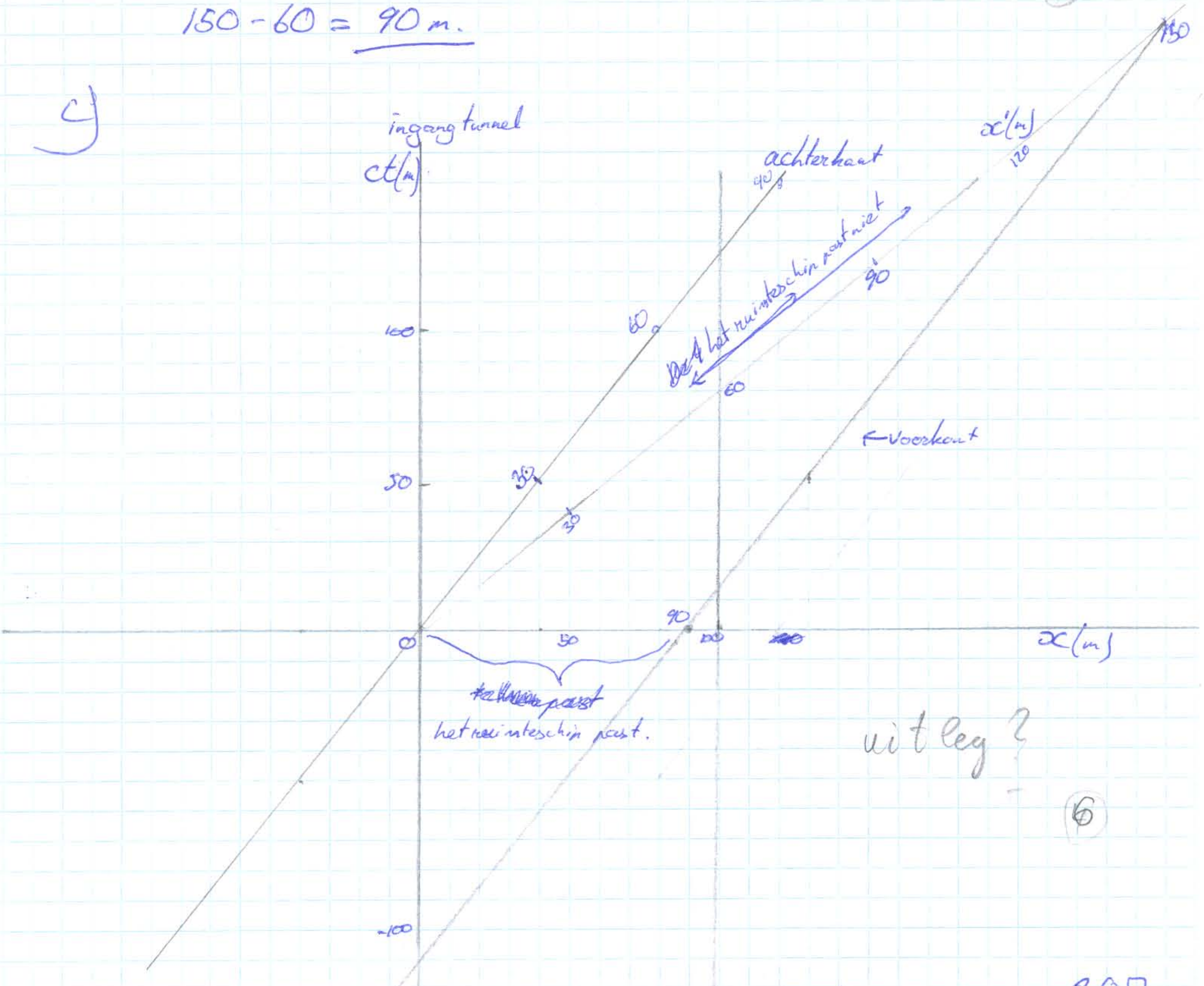
c)



uitleg?

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d) in het T stelsel

$$t = \frac{s}{v} = 10 \times \frac{5}{4} = 12{,}5 \text{ m}$$

$$t' = \Gamma(t - \beta x) \quad \text{met } x = 100$$

$$= \frac{5}{3}\left(12{,}5 - \frac{4}{5} \cdot 100\right) = -112{,}5 \text{ m}$$

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Opgave 3 Koplamp effect.

a) er geldt $V_y = \frac{V_y' \sqrt{1-V^2}}{1+\beta V_x}$ met $\Gamma = \frac{1}{\sqrt{1-V^2}}$

$V_y' = \sin\phi'$

$V_y = \sin\phi$

$V_x = \cos\phi'$

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Hieruit volgt

$$\sin\phi = \frac{\sin\phi'}{\Gamma(1+\beta\cos\phi')}$$

b) $\left.\begin{aligned}\beta &= \frac{3}{5} \\ \Gamma &= \frac{1}{\sqrt{1-\beta^2}}\end{aligned}\right\} \Gamma = \frac{5}{4}$

$$\sin\phi = \frac{4}{5}\,\frac{\sin 30}{1+0{,}6\cos 30}$$

$\sin 30 = \frac{1}{2}$

$\cos 30 = \frac{1}{2}\sqrt{3}$

$$= \frac{2}{5}\,\frac{1}{(1+0{,}3\sqrt{3}} = 0{,}26$$

$$\Rightarrow \underline{\phi = 15^\circ}$$

c) De helft van al het licht $\Rightarrow \phi' = 90^\circ$

$$\sin\phi = \frac{\sin\phi'}{\Gamma(1+\beta\cos\phi')}$$

$\sin 90^\circ = 1$

$\cos 90^\circ = 0$

$$= \frac{1}{\Gamma}$$

$\Gamma = \frac{1}{\sqrt{1+0{,}99^2}} = 7{,}09$

$$= 0{,}14$$

$$\Rightarrow \underline{\phi = 8{,}1^\circ}$$

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Opgave 4 $10 + 10 + 5 = 25$

Inelastische botsing

a) Scenario 1

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$$\begin{pmatrix} m \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \Gamma_v m \\ \Gamma_v m\beta \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \Gamma_n M \\ \Gamma_n M\beta' \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{array}{ll} (\Gamma_v + 1)m = \Gamma_n M & (1) \\ \Gamma_v m\beta = \Gamma_n M\beta' & (2) \end{array}$$

Γ_v = "Voor de botsing"
Γ_n = "Na de botsing"

$$\cancel{\frac{(1)}{(2)}} \quad \frac{(2)}{(1)} \Rightarrow \frac{\beta\Gamma_v}{\Gamma_v+1} = \beta'$$

$$\frac{\sqrt{\Gamma_v^2 - 1}}{\Gamma_v + 1} = \beta'$$

$$\sqrt{\frac{(\Gamma_v+1)(\Gamma_v-1)}{(\Gamma_v+1)(\Gamma_v+1)}} = \boxed{\sqrt{\frac{\Gamma_v - 1}{\Gamma_v + 1}} = \beta'}$$

$$\Gamma_n = \frac{1}{\sqrt{1-\beta'^2}}$$

$$= \frac{1}{\sqrt{\frac{(\Gamma_v+1)-(\Gamma_v-1)}{\Gamma_v+1}}}$$

$$= \sqrt{\frac{\Gamma_v+1}{2}}$$

$$(1) \Rightarrow M = \frac{\Gamma_v + 1}{\Gamma_n} \cdot m$$

$$= \sqrt{\frac{2}{\Gamma_v+1}} \times \Gamma_v + 1 \cdot m$$

$$\boxed{= m\sqrt{2(\Gamma_v+1)} = M}$$

Scenario 2

$$\begin{pmatrix} \Gamma_v m \\ \frac{\Gamma_v m\beta}{2} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \Gamma_v m \\ -\frac{\Gamma_v m\beta}{2} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \Gamma_n M \\ \Gamma_n M\beta' \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{array}{l} (1)\ 2\Gamma_v \cdot m = \Gamma_n \cdot M \\ (2)\ \Gamma_n \cdot M \cdot \beta' = 0 \end{array}$$

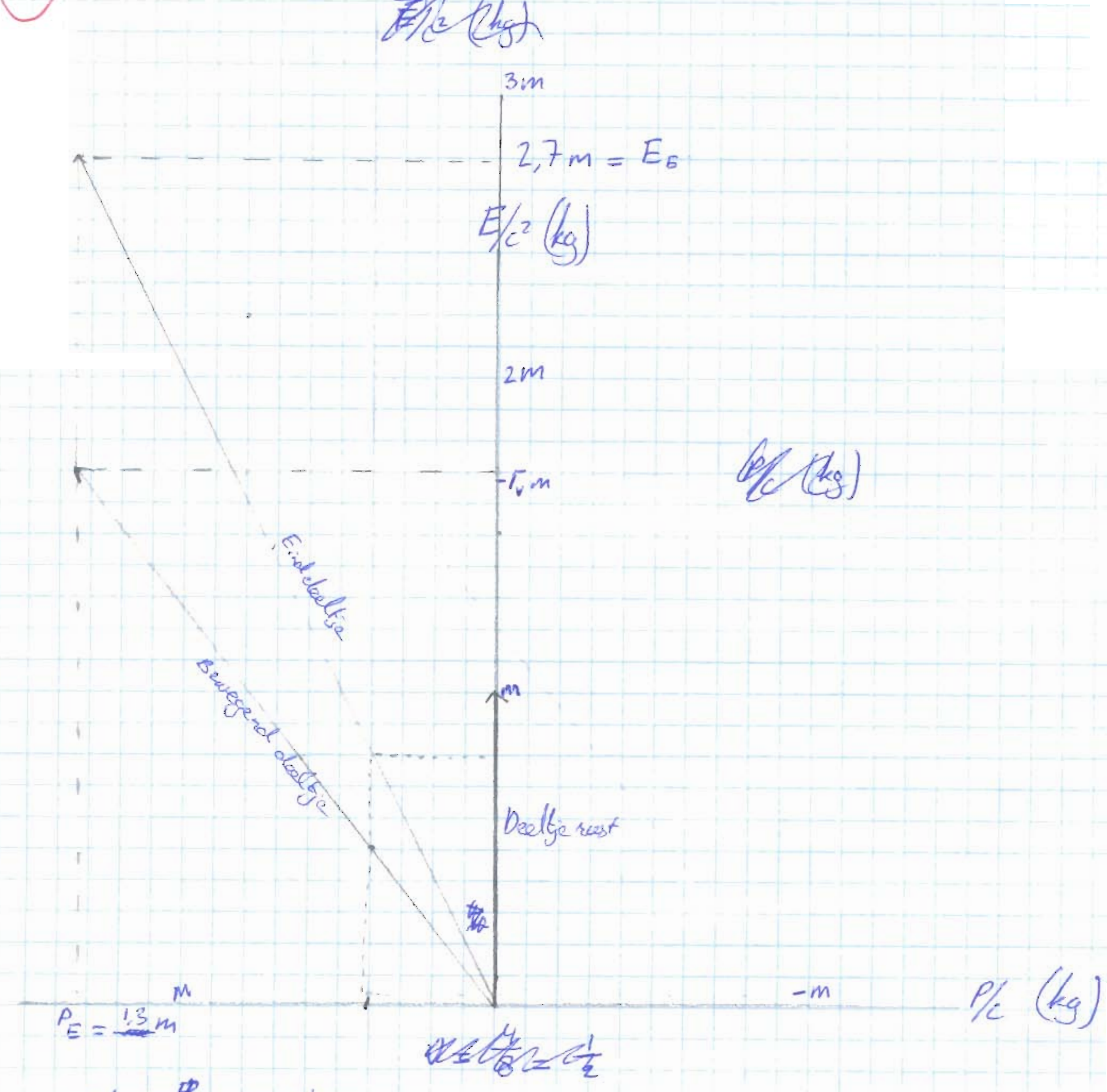
$$(2) \Rightarrow \boxed{\beta' = 0} \Rightarrow \Gamma_n = 1$$

$$(1) \Rightarrow \boxed{M = 2\Gamma_v \cdot m}$$

b] Senario 1

$$\beta = \frac{4}{5} \Rightarrow \Gamma_v = \frac{5}{3}$$

Deeltje rust   Deeltje 2   Deeltje na botsing



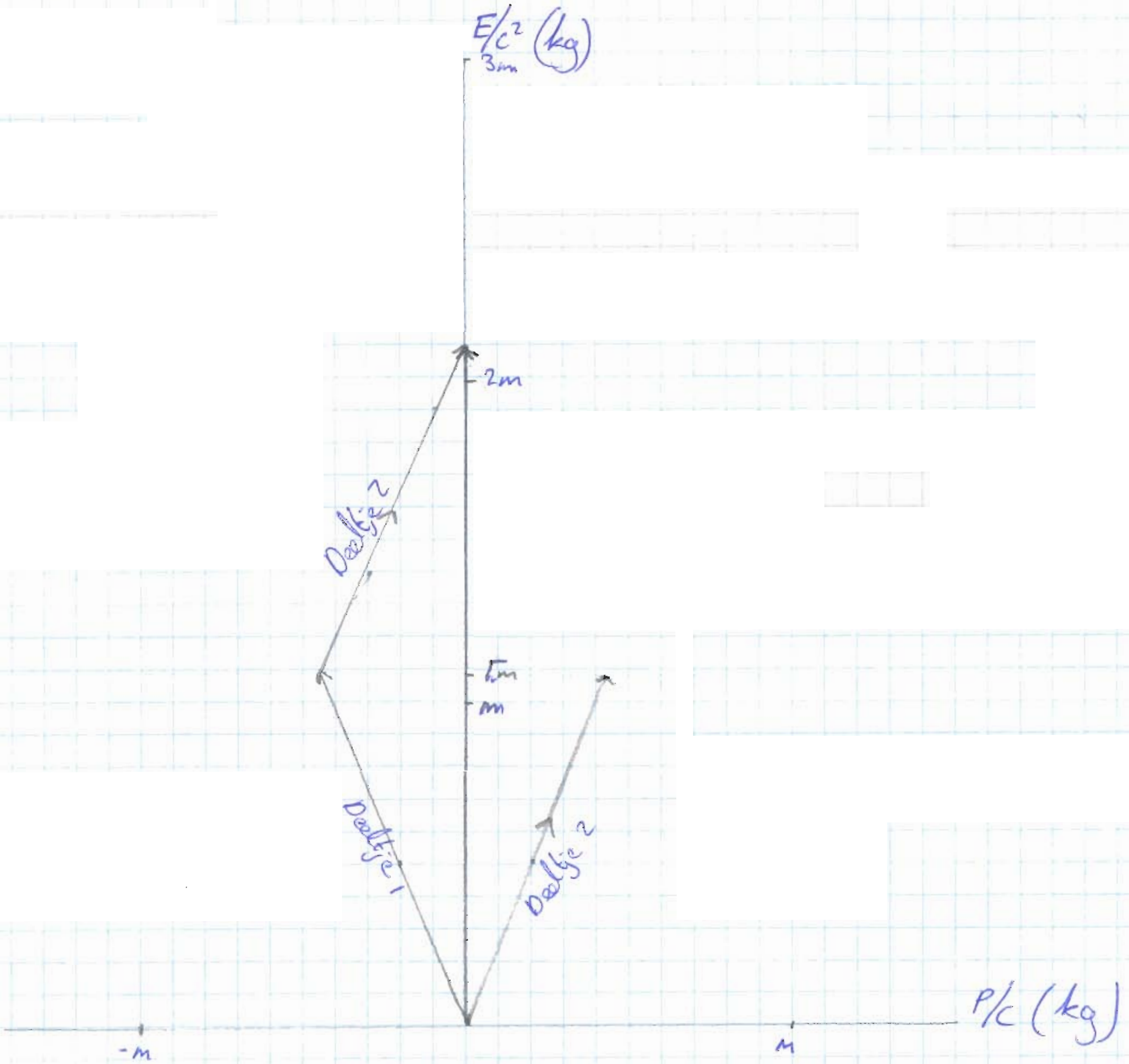
$$\beta' = \frac{p_E}{E_E} \approx \frac{1}{2}$$

$$\Gamma_n = 1{,}15$$

$$M = \frac{E_E}{\Gamma} = \frac{2{,}7m}{1{,}15} = \underline{2{,}3\ m}$$

Senario 2.

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{2}{5}\right)^2}} = 1,1$$



$$\left.\begin{array}{l} E_E = 2,1\,m \\ P_E = 0 \end{array}\right\} \beta' = \frac{P_E}{E_E} = 0 \Rightarrow \gamma_n = 1$$

$$M = \frac{E_E}{\gamma} \approx \underline{2,1\,m}$$ (Dit moet eigenlijk 2,2 m zijn maar door slordig tekenen is de afwijking groot)

c) omdat de snelheden waarmee de deeltje tegen elkaar botsen niet gelijk is. (Rekening houdend met het feit dat het verschillende stelsels zijn)

In Senario 2 is de snelheid van het ene deeltje t.o.v. het andere gegeven door

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$$V = \frac{\frac{1}{2}\beta + \frac{1}{2}\beta}{1 + \frac{1}{4}\beta^2}$$

$$= \frac{\beta}{1 + \frac{1}{4}\beta^2} \neq \beta$$

Bij hoge snelheden is dit wel degelijk anders dan de snelheid gegeven in Senario 1